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# Prediction of Friction and Heat Transfer for Viscoelastic Fluids in Turbulent Pipe Flow[1]

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Experimental measurements of the friction factor and the dimensionless heat-transfer *j*-factor were carried out for the turbulent pipe flow of viscoelastic aqueous solutions of polyacrylamide. The studies covered a wide range of variables including polymer concentration, polymer and solvent chemistry, pipe diameter, and flow rate. Degradation effects were also studied. It is concluded that the friction factor and the dimensionless heat transfer are functions only of the Reynolds number, the Weissenberg number, and the dimensionless distance, provided that the rheology of the flowing fluid is used.

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**KEY WORDS:** non-Newtonian fluid; polyacrylamide; polymer; rheology; viscoelasticity.

## 1. INTRODUCTION

The prediction of the friction-factor and heat-transfer behavior of Newtonian fluids in turbulent pipe flow is well established and may be found in current textbooks and handbooks in heat transfer [1–3]. The fully developed friction factor for isothermal flow may be predicted by the following equations:[3]

$$f = 0.079\, Re^{-0.25}, \qquad 5{,}000 < Re < 30{,}000 \tag{1}$$

$$f = 0.046\, Re^{-0.2}, \qquad 30{,}000 < Re < 100{,}000 \tag{2}$$

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[3] For an explanation of symbols, see Nomenclature.

The fully developed turbulent heat-transfer coefficient for constant property flow (i.e., small temperature differences) may be predicted by the following equation, provided that the Prandtl number ranges from 0.7 to 120:

$$j_H = St Pr^{2/3} = 0.023\,Re^{-0.2}, \qquad 10{,}000 < Re < 120{,}000 \tag{3}$$

Generally speaking the hydrodynamic and thermal entrance lengths are relatively short for Newtonian fluids with Prandtl numbers above 0.7, especially for a sharp-edge entrance. In general, fully established conditions prevail beyond a distance of 50 hydraulic diameters from the entrance. In arriving at the above correlations no consideration is given to the molecular structure of the working fluid, be it air, water, oil, or any other Newtonian fluid.[4] The macroscopic properties such as the Prandtl number, density, conductivity, and viscosity of the working fluid are determined, and when used in conjunction with the other relevant data such as the flow rate and pipe diameter, the heat transfer and pressure drop are predicted.

It is proposed that the same approach be used in dealing with aqueous polymer solutions such as polyacrylamide and polyethylene oxide. It is well known that such fluids are viscoelastic and that the pressure drop and heat transfer are dependent on the same variables which affect Newtonian fluids, but in addition, the behavior depends on the polymer concentration, on the chemistry of the polymer and the solvent, and on the state of degradation of the polymer. A brief review of the influence of these new variables is presented and a procedure proposed for developing predictive equations for such viscoelastic fluids.

## 2. INFLUENCE OF POLYMER CONCENTRATION

The steady shear viscosity of an aqueous polymer solution increases with an increase in the polymer concentration as shown in Fig. 1. At dilute

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[4] Except for liquid metals which have very low Prandtl numbers, of the order of 0.01.

**Table I.** Comprehensive Chemical Analysis of Chicago Tap Water[a]

| | | | | wppm | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Tap water | pH | Hardness | Total alkalinity | $SO_4^{2-}$ | $Cl^-$ | $Ca^{2+}$ | $Mg^{2+}$ | $K^+$ | $Na^+$ |
| Chicago | 8.0 | 130 | 106 | 25 | 11 | 37 | 9 | 1.0 | 5.3 |

[a] Data supplied by Water Purification Laboratory, Chicago.

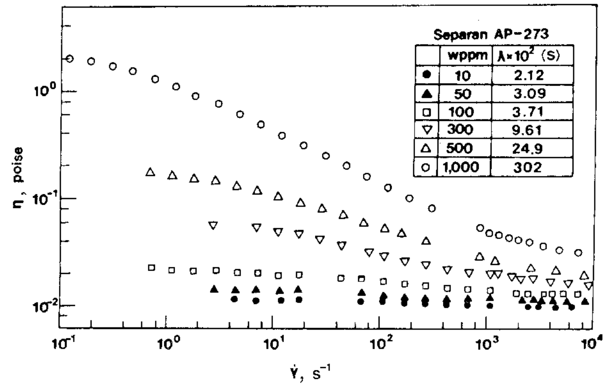

Fig. 1. $\eta$ vs $\gamma$ of Separan solutions with various concentrations.

concentrations, of the order of 10 parts per million by weight (wppm), the viscosity of the solution is not distinguishable from the solvent, which in the present study is Chicago tap water (see Table I for the chemical composition). As the concentration increases the viscosity increases, especially at lower shear rates.

The corresponding fully developed friction factors for turbulent pipe flow are shown in Fig. 2 as a function of the apparent Reynolds number, $\rho Vd/\eta$, where $\eta$ is the steady shear viscosity corresponding to the local wall shearing stress. It is clear that there is a substantial decrease in the friction factor even at 10 wppm and that the decrease in friction factor with

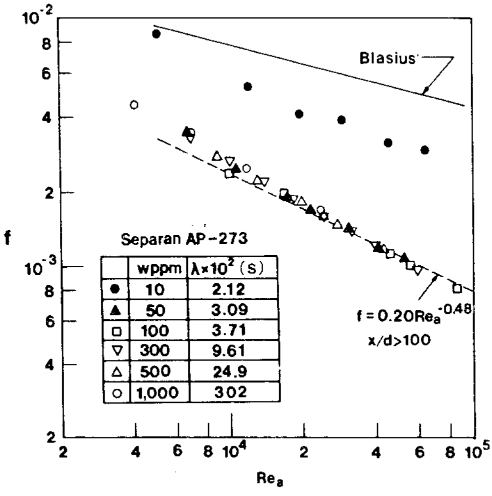

Fig. 2. $f$ vs $Re_a$ of Separan solutions with various concentrations.

increasing polymer concentration continues up to 50 wppm. If the polymer concentration is increased above 50 wppm, no further changes occur in the friction factor. This behavior was first reported by Virk [4] and the asymptotic value is often called Virk's maximum drag reduction asymptote.

The hydrodynamic entrance length for aqueous polyacrylamide solutions is a function of the polymer concentration, ranging from the Newtonian values of $L_e/d \simeq 50$ for dilute concentrations up to $L_e/d \simeq 100$ for concentrations which yield asymptotic friction-factor values [5]. All of the data shown in Fig. 2 were taken in the fully developed flow region.

The heat-transfer behavior of the aqueous polyacrylamide solution is given in Fig. 3 in the form of $j_H$ vs $Re_a$. The dimensionless Colburn heat-transfer factor $j_H$ is defined as $St \cdot Pr^{2/3}$ and is relatively independent of the Prandtl number for *Newtonian* fluids. It may be noted from Fig. 3 that the dimensionless heat-transfer coefficient decreases with increasing polymer concentration and that a heat-transfer asymptote is ultimately reached at a concentration of 500 to 1000 wppm. This is in marked contrast to the friction factor, which required only 50 wppm to reach an asymptotic value. Another major difference is in the thermal entrance length. Whereas the hydrodynamic entrance length corresponding to the asymptotic friction factor is of the order of 100 pipe diameters, it requires approximately 430 pipe diameters to reach fully developed heat-transfer conditions for the

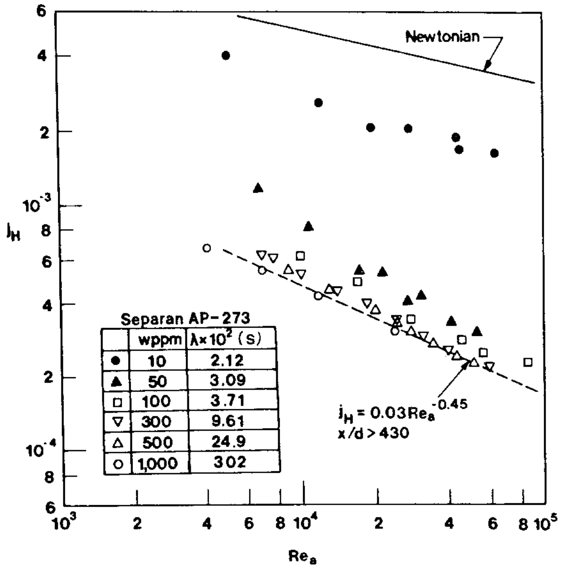

**Fig. 3.** $j_H$ vs $Re_a$ of Separan solutions with various concentrations.

higher concentration of polyacrylamide. All of the reported measurements correspond to this condition.

A comparison of Figs. 2 and 3 also brings out the fact that the decrease in heat-transfer coefficient and friction factor are of the same order for dilute solutions, but at higher concentrations the heat-transfer reduction is greater than the friction-factor reduction.

## 3. INFLUENCE OF POLYMER DEGRADATION

When an aqueous polymer solution is subject to shearing stresses in the course of being circulated through a flow system, the polymer bonds are sheared. This process, called degradation, influences the properties of the aqueous polymer solution, which in turn affects the pressure-drop and heat-transfer behavior. Figure 4 shows the measured friction factor and dimensionless heat-transfer coefficient, $j_H$, as a function of hours of circulation for a 1000 wppm aqueous polyacrylamide solution flowing turbulently through a 1.30-cm tube at a fixed Reynolds number of 20,000. The data are taken at an $x/d$ of 430 and closely approximate fully developed conditions. Also shown as solid lines are the asymptotic values. It is clear that the friction factor maintains its minimum value for approximately 40 h before it begins to increase dramatically. In contrast, the heat transfer increases immediately from the beginning of circulation. Samples of the fluid are removed from the flow loop at different hours of circulation and

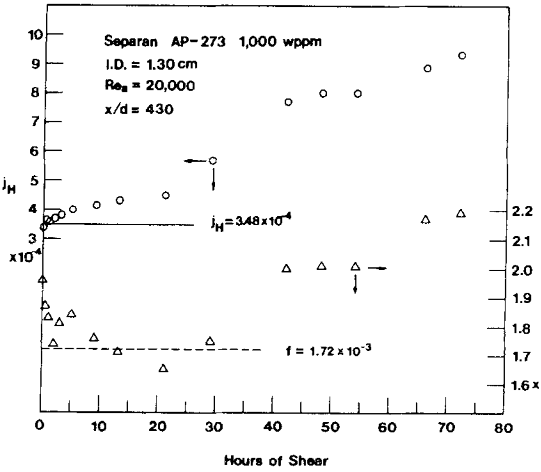

Fig. 4. $f$ or $j_H$ vs hours of shear of Separan 1000 wppm solution.

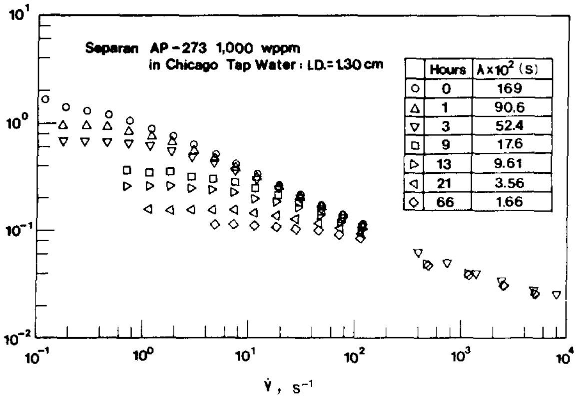

**Fig. 5.** $\eta$ vs $\gamma$ of Separan 1000 wppm solution for various hours of shear.

the steady shear viscosity is measured, yielding the values shown in Fig. 5. The major influence of degradation is seen to occur at the lower shear rates, where the viscosity decreases with degradation as the molecular bonds are sheared. Little effect of degradation is seen at the higher shear rates. This brings out the fact that capillary measurements taken at higher shear rates will not reveal any significant change in the viscosity resulting from degradation.

## 4. INFLUENCE OF SOLVENT CHEMISTRY

A number of studies have brought out the fact that the solvent chemistry plays a major role in determining the rheology of an aqueous polymer solution [6–8]. This is demonstrated in Fig. 6, which shows the

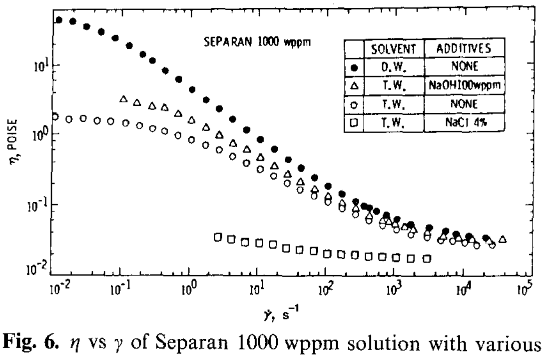

**Fig. 6.** $\eta$ vs $\gamma$ of Separan 1000 wppm solution with various additives: basic solvents are distilled water (D.W.) and tap water (T.W.).

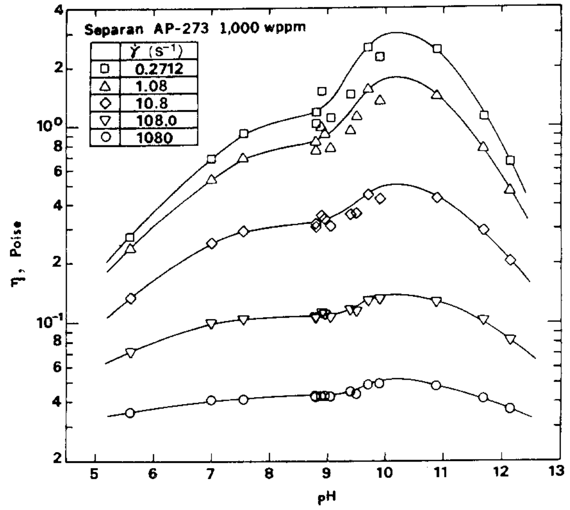

Fig. 7. $\eta$ vs pH of Separan 1000 wppm solution with various additives.

steady shear viscosity as a function of the shear rate for 1000 wppm of polyacrylamide with water as the solvent. The dramatic effect on the low shear-rate viscosity of modifying the water chemistry is brought out clearly. Figure 7 presents the same results as a function of the pH of the solvent, which suggests that the maximum steady shear viscosity for a fixed shear rate occurs at a pH approximately equal to 10 for the aqueous polyacrylamide solution.

The influence of solvent chemistry on the dimensionless heat-transfer factor, $j_H$, is shown in Fig. 8. Three solutions of 20 wppm of polyacrylamide in Chicago tap water were studied: (1) Chicago tap water without additives, (2) Chicago tap water plus 20 wppm NaOH, and (3) Chicago tap water plus 100 wppm NaOH. It is obvious that the addition of sodium hydroxide results in substantial decreases in the heat transfer, with the 100 wppm NaOH solution showing the larger reduction. There is clear evidence of degradation at the higher Reynolds numbers but this does not obscure the conclusion that solvent chemistry plays a major role in the rheological, hydrodynamic, and heat-transfer behavior of polymer solution.

It should be noted that the chemistry of the solute (e.g., the polymer) also affects the rheology, and the friction factor and heat transfer. Since the molecular weight distribution of commercial polymers varies from batch to batch, the resulting zero shear-rate viscosities of concentrated aqueous solutions may vary by a factor of 10 from polymer batch to polymer batch for a given solvent [9].

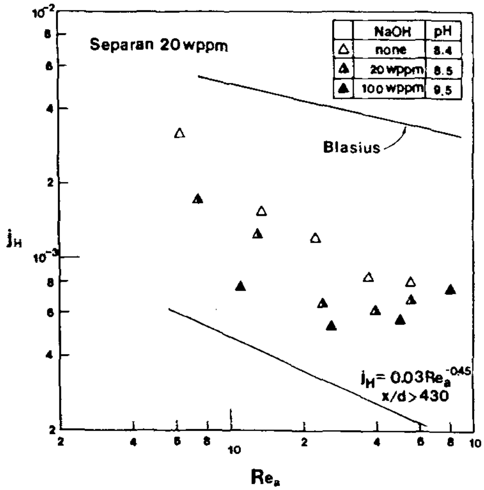

Fig. 8. $j_H$ vs $Re_a$ of Separan solutions.

## 5. PROPOSED PROCEDURE FOR PREDICTING FRICTION FACTOR AND HEAT TRANSFER

It is proposed that the fully developed friction factor and dimensionless heat-transfer coefficient are functions of the dimensionless Reynolds number, $Re_a$, and a second dimensionless quantity which accounts for the effects of the fluid elasticity, the Weissenberg number, defined as $\lambda V/d$. This characteristic time of the fluid $\lambda$ can be determined in a number of ways and its absolute magnitude depends on the procedure used. The approach used in this study was to take the value of the characteristic time predicted by the sophisticated Carreau-Bird model B as the standard to be compared with the values predicted by a number of generalized Newtonian models. It was found that the Powell-Eyring model yielded values which were in good agreement with the Carreau-Bird model B. As a consequence the Powell-Eyring characteristic time was used since it offers a relatively simple method for determining $\lambda$. The measured steady shear viscosity data were fitted to the Powell-Eyring equation,

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty)\left(\frac{\sinh^{-1}\lambda\dot{\gamma}}{\lambda\dot{\gamma}}\right) \tag{4}$$

where $\lambda$ corresponds to the value yielding the best fit to the data.

Experiments were conducted over a wide range of operating conditions, including the effects of polymer concentration, solvent chemistry,

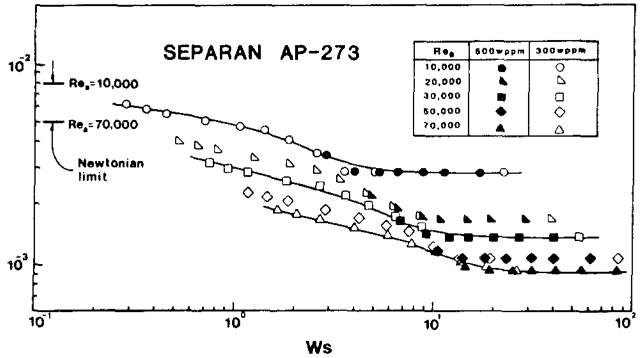

Fig. 9. Generalized $f$ vs $Ws$ (with $Re_a$ as parameter) of Separan solutions.

and degradation, in a once-through flow mode as well as in a recirculating system. Three different pipe diameters, ranging from 1.30 to 2.25 cm, were used. If the rheology of the *flowing fluid* is used, it was found empirically that the measured fully developed friction-factor data were a function only of the Reynolds number $Re_a$ and the Weissenberg number $Ws$. This of course requires continuous monitoring of the fluid if it is circulated, with the heat transfer and friction factor at any time being a function of the rheology of the fluid at that same time.

An example of the results obtained is shown in Fig. 9, where it may be seen that the fully established friction factor, $f$, is a function only of the

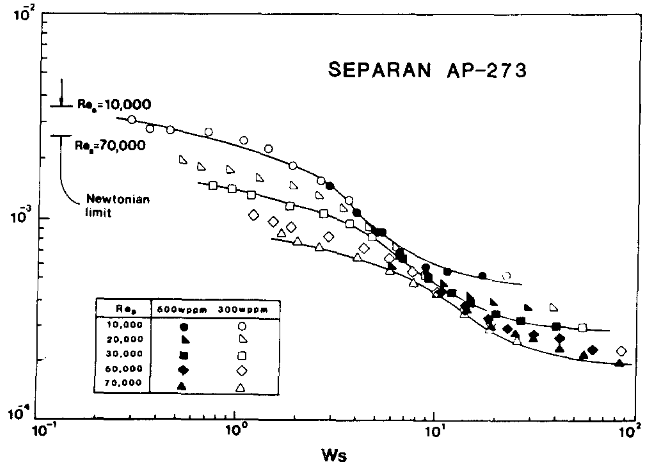

Fig. 10. Generalized $j_H$ vs $Ws$ (with $Re_a$ as parameter) of Separan solutions.

Weissenburg and Reynolds numbers. Similar results were obtained for the fully developed heat-transfer factor, $j_H$, as shown in Fig. 10. For the aqueous polyacrylamide solution it is concluded that, in general,

$$f = f(x/d, Re_a, Ws) \tag{5}$$

and

$$j_H = j_H(x/d, Re_a, Ws) \tag{6}$$

It is not suggested that the functional relationship found for aqueous polyacrylamide solution is valid for other polymer solutions. However, it is proposed that the approach taken here should apply to other polymer solutions.

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## NOMENCLATURE

| | |
|---|---|
| $c_p$ | Specific heat of fluid, $J \cdot kg^{-1} \cdot K^{-1}$ |
| $d$ | Diameter of tube, m |
| $f$ | Fanning friction factor, $\tau_w/(\rho V^2/2)$ |
| $h$ | Convective heat-transfer coefficient, $q_w/(T_w - T_b)$, $W \cdot m^{-2} \cdot K^{-1}$ |
| $k$ | Thermal conductivity of fluid, $W \cdot m^{-1} \cdot K^{-1}$ |
| $j_H$ | Heat-transfer $j$-factor, $StPr_a^{2/3}$ |
| $L_e$ | Entrance length, m |
| $Nu$ | Nusselt number, $hd/k$ |
| $Pr_a$ | Prandtl number based on apparent viscosity at the wall, $\eta c_p/k$ |
| $q_w$ | Heat flux at the wall, $W \cdot m^{-2}$ |
| $Re_a$ | Reynolds number based on apparent viscosity at the wall, $\rho Vd/\eta$ |
| $St$ | Stanton number, $Nu/(Re_a Pr_a)$ |
| $T$ | Temperature, K |
| $T_b$ | Bulk temperature of fluid, K |
| $T_w$ | Inside-wall temperature, K |
| $V$ | Average velocity, $m \cdot s^{-1}$ |
| $Ws$ | Weissenberg number, $\lambda V/d$ |
| $x$ | Axial coordinate, m |

## Greek Symbols

| | |
|---|---|
| $\dot{\gamma}$ | Shear rate, $s^{-1}$ |
| $\eta$ | Apparent viscosity evaluated at the wall, P[5] |
| $\eta_0$ | Zero shear-rate viscosity, P[5] |
| $\eta_\infty$ | Apparent viscosity at infinite shear rate, P[5] |
| $\lambda$ | Characteristic time of fluid, s |
| $\rho$ | Density of fluid, $kg \cdot m^{-3}$ |
| $\tau_w$ | Wall shear stress, $N \cdot m^{-2}$ |

[5] One poise (P) = 0.1 Pa · s.

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